Limit Rules

Useful rules for finding limits:

Limit Rules

Useful rules for finding limits:

In the following rules assume $\mathbf{k = constant}$

Limit Rules

Useful rules for finding limits:

In the following rules assume $\mathbf{k = constant}$

$$\lim_{x \longrightarrow c} k = k$$

Limit Rules

Useful rules for finding limits:

In the following rules assume **k = constant**

$$\lim_{x \longrightarrow c} k = k$$

$$\lim_{x \longrightarrow c} kf(x) = k \lim_{x \longrightarrow c} f(x)$$

Limit Rules

Useful rules for finding limits:
In the following rules assume **k = constant**

$$\lim_{x \longrightarrow c} k = k$$
$$\lim_{x \longrightarrow c} kf(x) = k \lim_{x \longrightarrow c} f(x)$$
$$\lim_{x \longrightarrow c} f(x) \pm g(x) = \lim_{x \longrightarrow c} f(x) \pm \lim_{x \longrightarrow c} g(x)$$

Limit Rules

Useful rules for finding limits:

In the following rules assume **k = constant**

$$\lim_{x \longrightarrow c} k = k$$

$$\lim_{x \longrightarrow c} kf(x) = k \lim_{x \longrightarrow c} f(x)$$

$$\lim_{x \longrightarrow c} f(x) \pm g(x) = \lim_{x \longrightarrow c} f(x) \pm \lim_{x \longrightarrow c} g(x)$$

$$\lim_{x \longrightarrow c} f(x) \cdot g(x) = \lim_{x \longrightarrow c} f(x) \cdot \lim_{x \longrightarrow c} g(x)$$

Limit Rules

Useful rules for finding limits:

In the following rules assume **k = constant**

$$\lim_{x \longrightarrow c} k = k$$

$$\lim_{x \longrightarrow c} kf(x) = k \lim_{x \longrightarrow c} f(x)$$

$$\lim_{x \longrightarrow c} f(x) \pm g(x) = \lim_{x \longrightarrow c} f(x) \pm \lim_{x \longrightarrow c} g(x)$$

$$\lim_{x \longrightarrow c} f(x) \cdot g(x) = \lim_{x \longrightarrow c} f(x) \cdot \lim_{x \longrightarrow c} g(x)$$

$$\lim_{x \longrightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \longrightarrow c} f(x)}{\lim_{x \longrightarrow c} g(x)_{(\neq 0)}}$$

Limit Rules

Useful rules for finding limits:

In the following rules assume $\mathbf{k = constant}$

$$\lim_{x \longrightarrow c} k = k$$

$$\lim_{x \longrightarrow c} kf(x) = k \lim_{x \longrightarrow c} f(x)$$

$$\lim_{x \longrightarrow c} f(x) \pm g(x) = \lim_{x \longrightarrow c} f(x) \pm \lim_{x \longrightarrow c} g(x)$$

$$\lim_{x \longrightarrow c} f(x) \cdot g(x) = \lim_{x \longrightarrow c} f(x) \cdot \lim_{x \longrightarrow c} g(x)$$

$$\lim_{x \longrightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \longrightarrow c} f(x)}{\lim_{x \longrightarrow c} g(x)_{(\neq 0)}}$$

$$\lim_{x \longrightarrow c} [f(x)^n] = \left[\lim_{x \longrightarrow c} f(x)\right]^n$$

Limit Rules

Useful rules for finding limits:

In the following rules assume **k = constant**

$$\lim_{x \longrightarrow c} k = k$$

$$\lim_{x \longrightarrow c} kf(x) = k \lim_{x \longrightarrow c} f(x)$$

$$\lim_{x \longrightarrow c} f(x) \pm g(x) = \lim_{x \longrightarrow c} f(x) \pm \lim_{x \longrightarrow c} g(x)$$

$$\lim_{x \longrightarrow c} f(x) \cdot g(x) = \lim_{x \longrightarrow c} f(x) \cdot \lim_{x \longrightarrow c} g(x)$$

$$\lim_{x \longrightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \longrightarrow c} f(x)}{\lim_{x \longrightarrow c} g(x)_{(\neq 0)}}$$

$$\lim_{x \longrightarrow c} [f(x)^n] = \left[\lim_{x \longrightarrow c} f(x)\right]^n$$

$$\lim_{x \longrightarrow c} P(x) = P(c)$$, P(x) is continuous around c, just plug-in

Limit Rules

example

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = ?$$

Limit Rules

example

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = ?$$

first try "limit of ratio $=$ ratio of limits rule",

Limit Rules

example

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = ?$$

first try "limit of ratio = ratio of limits rule",

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} =$$

Limit Rules

example

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = ?$$

first try "limit of ratio = ratio of limits rule",

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = \frac{\lim_{x \longrightarrow 3} x^2 - 9}{\lim_{x \longrightarrow 3} x - 3} =$$

Limit Rules

example

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = ?$$

first try "limit of ratio = ratio of limits rule",

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \frac{\lim_{x \to 3} x^2 - 9}{\lim_{x \to 3} x - 3} = \frac{0}{0}$$

Limit Rules

example

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = ?$$

first try "limit of ratio = ratio of limits rule",

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = \frac{\lim_{x \longrightarrow 3} x^2 - 9}{\lim_{x \longrightarrow 3} x - 3} = \frac{0}{0}$$

$\frac{0}{0}$ is called an indeterminant form.

Limit Rules

example

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = ?$$

first try "limit of ratio = ratio of limits rule",

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = \frac{\lim_{x \longrightarrow 3} x^2 - 9}{\lim_{x \longrightarrow 3} x - 3} = \frac{0}{0}$$

$\frac{0}{0}$ is called an indeterminant form. When you reach an indeterminant form you need to try someting else.

Limit Rules

example

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = ?$$

first try "limit of ratio = ratio of limits rule",

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = \frac{\lim_{x \longrightarrow 3} x^2 - 9}{\lim_{x \longrightarrow 3} x - 3} = \frac{0}{0}$$

$\frac{0}{0}$ is called an indeterminant form. When you reach an indeterminant form you need to try someting else.

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} =$$

Limit Rules

example

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = ?$$

first try "limit of ratio = ratio of limits rule",

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \frac{\lim_{x \to 3} x^2 - 9}{\lim_{x \to 3} x - 3} = \frac{0}{0}$$

$\frac{0}{0}$ is called an indeterminant form. When you reach an indeterminant form you need to try someting else.

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \lim_{x \to 3} \frac{(x - 3)(x + 3)}{(x - 3)} =$$

Limit Rules

example

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = ?$$

first try "limit of ratio = ratio of limits rule",

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = \frac{\lim_{x \longrightarrow 3} x^2 - 9}{\lim_{x \longrightarrow 3} x - 3} = \frac{0}{0}$$

$\frac{0}{0}$ is called an indeterminant form. When you reach an indeterminant form you need to try someting else.

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = \lim_{x \longrightarrow 3} \frac{(x-3)(x+3)}{(x-3)} = \lim_{x \longrightarrow 3} (x+3) =$$

Limit Rules

example

$$\lim_{x\longrightarrow 3} \frac{x^2-9}{x-3} = ?$$

first try "limit of ratio = ratio of limits rule",

$$\lim_{x\longrightarrow 3} \frac{x^2-9}{x-3} = \frac{\lim_{x\longrightarrow 3} x^2-9}{\lim_{x\longrightarrow 3} x-3} = \frac{0}{0}$$

$\frac{0}{0}$ is called an indeterminant form. When you reach an indeterminant form you need to try someting else.

$$\lim_{x\longrightarrow 3} \frac{x^2-9}{x-3} = \lim_{x\longrightarrow 3} \frac{(x-3)(x+3)}{(x-3)} = \lim_{x\longrightarrow 3}(x+3) = 3+3 =$$

Limit Rules

example

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = ?$$

first try "limit of ratio = ratio of limits rule",

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = \frac{\lim_{x \longrightarrow 3} x^2 - 9}{\lim_{x \longrightarrow 3} x - 3} = \frac{0}{0}$$

$\frac{0}{0}$ is called an indeterminant form. When you reach an indeterminant form you need to try someting else.

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = \lim_{x \longrightarrow 3} \frac{(x-3)(x+3)}{(x-3)} = \lim_{x \longrightarrow 3}(x+3) = 3 + 3 = 6$$

Limit Rules

example

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = ?$$

first try "limit of ratio = ratio of limits rule",

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = \frac{\lim_{x \longrightarrow 3} x^2 - 9}{\lim_{x \longrightarrow 3} x - 3} = \frac{0}{0}$$

$\frac{0}{0}$ is called an indeterminant form. When you reach an indeterminant form you need to try someting else.

$$\lim_{x \longrightarrow 3} \frac{x^2 - 9}{x - 3} = \lim_{x \longrightarrow 3} \frac{(x - 3)(x + 3)}{(x - 3)} = \lim_{x \longrightarrow 3} (x + 3) = 3 + 3 = 6$$

Indeterminant does not mean the limit does not exist.

Limit Rules

example

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = ?$$

first try "limit of ratio = ratio of limits rule",

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \frac{\lim_{x \to 3} x^2 - 9}{\lim_{x \to 3} x - 3} = \frac{0}{0}$$

$\frac{0}{0}$ is called an indeterminant form. When you reach an indeterminant form you need to try someting else.

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \lim_{x \to 3} \frac{(x - 3)(x + 3)}{(x - 3)} = \lim_{x \to 3} (x + 3) = 3 + 3 = 6$$

Indeterminant does not mean the limit does not exist.
It just means that the method you tried did not tell you anything and you need to try another method.

Limit Rules

indeterminant forms

- when taking limits, $\frac{\mathbf{0}}{\mathbf{0}}$, $\frac{\infty}{\infty}$ and $\infty \cdot \mathbf{0}$ are called **indeterminant forms**.

Limit Rules

indeterminant forms

- when taking limits, $\frac{\mathbf{0}}{\mathbf{0}}$, $\frac{\infty}{\infty}$ and $\infty \cdot \mathbf{0}$ are called **indeterminant forms**.
- indeterminant form means that the method you used to find the limit led you to a point where you cannot make any conclusions about the limit.

Limit Rules

indeterminant forms

- when taking limits, $\frac{\mathbf{0}}{\mathbf{0}}$, $\frac{\infty}{\infty}$ and $\infty \cdot \mathbf{0}$ are called **indeterminant forms**.
- indeterminant form means that the method you used to find the limit led you to a point where you cannot make any conclusions about the limit.
- when you reach an indeterminant form, you must try another method to determine the limit. This usually means to first use an algebra trick and then continue finding the limit.

Limit Rules

indeterminant forms

- when taking limits, $\frac{0}{0}$, $\frac{\infty}{\infty}$ and $\infty \cdot 0$ are called **indeterminant forms**.
- indeterminant form means that the method you used to find the limit led you to a point where you cannot make any conclusions about the limit.
- when you reach an indeterminant form, you must try another method to determine the limit. This usually means to first use an algebra trick and then continue finding the limit.
- indeterminant does not mean that the limit cannot be determined. It only means that the method you used did not work. You must try another method to determine the limit. In some cases this could just mean using the table method.

Limit Rules

example

$$\lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2} = ?$$

Limit Rules

example

$$\lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2} = ?$$

First try "limit of ratio = ratio of limits rule"

Limit Rules

example

$$\lim_{x \to 4} \frac{x-4}{\sqrt{x}-2} = ?$$

First try "limit of ratio = ratio of limits rule" and will again get $\frac{0}{0}$

Limit Rules

example

$$\lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2} = ?$$

First try "limit of ratio = ratio of limits rule" and will again get $\frac{0}{0}$ an indeterminant form.

Limit Rules

example

$$\lim_{x \to 4} \frac{x-4}{\sqrt{x}-2} = ?$$

First try "limit of ratio = ratio of limits rule" and will again get $\frac{0}{0}$ an indeterminant form. Try an algebra trick:

Limit Rules

example

$$\lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2} = ?$$

First try "limit of ratio = ratio of limits rule" and will again get $\frac{0}{0}$ an indeterminant form. Try an algebra trick:

$$\lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2} = \lim_{x \to 4} \frac{(\sqrt{x})^2 - 2^2}{(\sqrt{x} - 2)} \tag{1}$$

$$= \lim_{x \to 4} \frac{(\sqrt{x} - 2)(\sqrt{x} + 2)}{(\sqrt{x} - 2)} \tag{2}$$

$$= \lim_{x \to 4} (\sqrt{x} + 2) \tag{3}$$

$$= (\sqrt{4} + 2) = 2 + 2 = 4 \tag{4}$$

Limit Rules

same example, new trick

$$\lim_{x\to4}\frac{x-4}{\sqrt{x}-2} = \lim_{x\to4}\frac{x-4}{(\sqrt{x}-2)}\cdot\frac{(\sqrt{x}+2)}{(\sqrt{x}+2)} \quad (5)$$

$$= \lim_{x\to4}\frac{(x-4)(\sqrt{x}+2)}{(\sqrt{x}-2)(\sqrt{x}+2)} \quad (6)$$

$$= \lim_{x\to4}\frac{(x-4)(\sqrt{x}+2)}{(x-4)} \quad (7)$$

$$= \lim_{x\to4}(\sqrt{x}+2) \quad (8)$$

$$= (\sqrt{4}+2) = 2+2 = 4 \quad (9)$$

Limit Rules

indeterminant form $\frac{0}{0}$

There are three cases for $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{0}{0}$

Limit Rules

indeterminant form $\frac{0}{0}$

There are three cases for $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{0}{0}$

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \begin{cases} 0 & \text{top goes to } 0 \text{ faster than bottom} \\ & \\ & \end{cases}$$

Limit Rules

indeterminant form $\frac{0}{0}$

There are three cases for $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{0}{0}$

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \begin{cases} 0 & \text{top goes to } 0 \text{ faster than bottom} \\ \text{constant} \neq 0 & \text{top and bottom go to } 0 \text{ at same rate} \\ & \end{cases}$$

Limit Rules

indeterminant form $\frac{0}{0}$

There are three cases for $\lim_{x \longrightarrow c} \frac{f(x)}{g(x)} = \frac{0}{0}$

$$\lim_{x \longrightarrow c} \frac{f(x)}{g(x)} = \begin{cases} 0 & \text{top goes to } 0 \text{ faster than bottom} \\ \text{constant} \neq 0 & \text{top and bottom go to } 0 \text{ at same rate} \\ \infty & \text{bottom goes to zero faster than top} \end{cases}$$

Limit Rules

indeterminant form $\frac{\infty}{\infty}$

There are three cases for $\lim_{x \to c} f(x) = \frac{\infty}{\infty}$

Limit Rules

indeterminant form $\frac{\infty}{\infty}$

There are three cases for $\lim_{x \to c} f(x) = \frac{\infty}{\infty}$

$$\lim_{x \to c} \frac{f(x)}{g(x)} =$$

Limit Rules

indeterminant form $\frac{\infty}{\infty}$

There are three cases for $\lim_{x \longrightarrow c} f(x) = \frac{\infty}{\infty}$

$$\lim_{x \longrightarrow c} \frac{f(x)}{g(x)} = \begin{cases} \infty & \text{top goes to } \infty \text{ faster than bottom} \\ & \end{cases}$$

Limit Rules

indeterminant form $\frac{\infty}{\infty}$

There are three cases for $\lim_{x \to c} f(x) = \frac{\infty}{\infty}$

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \begin{cases} \infty & \text{top goes to } \infty \text{ faster than bottom} \\ \text{constant} \neq 0 & \text{top and bottom go to } \infty \text{ at same rate} \end{cases}$$

Limit Rules

indeterminant form $\frac{\infty}{\infty}$

There are three cases for $\lim_{x \longrightarrow c} f(x) = \frac{\infty}{\infty}$

$$\lim_{x \longrightarrow c} \frac{f(x)}{g(x)} = \begin{cases} \infty & \text{top goes to } \infty \text{ faster than bottom} \\ \text{constant} \neq 0 & \text{top and bottom go to } \infty \text{ at same rate} \\ 0 & \text{bottom goes to } \infty \text{ faster than top} \end{cases}$$

Limit Rules

indeterminant form $\frac{\infty}{\infty}, \frac{0}{0}$

Exercise: to demonstrate the preveous six results evaluate the following six limits by using the table method for each:

1. $\lim_{x \to \infty} \frac{x}{x^2}$

Use powers of 10 for x

Limit Rules

indeterminant form $\frac{\infty}{\infty}$, $\frac{0}{0}$

Exercise: to demonstrate the preveous six results evaluate the following six limits by using the table method for each:

1. $\lim_{x \to \infty} \frac{x}{x^2}$
2. $\lim_{x \to \infty} \frac{x^2}{x}$

Use powers of 10 for x

Limit Rules

indeterminant form $\frac{\infty}{\infty}, \frac{0}{0}$

Exercise: to demonstrate the preveous six results evaluate the following six limits by using the table method for each:

1. $\lim_{x \to \infty} \frac{x}{x^2}$
2. $\lim_{x \to \infty} \frac{x^2}{x}$
3. $\lim_{x \to \infty} \frac{x^2}{x^2}$

Use powers of 10 for x

Limit Rules

indeterminant form $\frac{\infty}{\infty}, \frac{0}{0}$

Exercise: to demonstrate the preveous six results evaluate the following six limits by using the table method for each:

1. $\lim_{x \to \infty} \frac{x}{x^2}$
2. $\lim_{x \to \infty} \frac{x^2}{x}$
3. $\lim_{x \to \infty} \frac{x^2}{x^2}$
4. $\lim_{x \to 0} \frac{x}{x^2}$

Use powers of $\mathbf{10}$ for $\mathbf{x}$

Limit Rules

indeterminant form $\frac{\infty}{\infty}$, $\frac{0}{0}$

Exercise: to demonstrate the preveous six results evaluate the following six limits by using the table method for each:

1. $\lim_{x \to \infty} \frac{x}{x^2}$
2. $\lim_{x \to \infty} \frac{x^2}{x}$
3. $\lim_{x \to \infty} \frac{x^2}{x^2}$
4. $\lim_{x \to 0} \frac{x}{x^2}$
5. $\lim_{x \to 0} \frac{x^2}{x}$

Use powers of $\mathbf{10}$ for $\mathbf{x}$

Limit Rules

indeterminant form $\frac{\infty}{\infty}, \frac{0}{0}$

Exercise: to demonstrate the preveous six results evaluate the following six limits by using the table method for each:

1. $\lim_{x\to\infty} \frac{x}{x^2}$
2. $\lim_{x\to\infty} \frac{x^2}{x}$
3. $\lim_{x\to\infty} \frac{x^2}{x^2}$
4. $\lim_{x\to 0} \frac{x}{x^2}$
5. $\lim_{x\to 0} \frac{x^2}{x}$
6. $\lim_{x\to 0} \frac{x^2}{x^2}$

Use powers of 10 for x