

$$f(x) = e^{-x^2}$$

$$f'(x) = \frac{d}{dx}(-x^2) \cdot e^{-x^2}$$

general exponential rule

$$f'(x) = -2x \cdot e^{-x^2}$$

$$f''(x) = \frac{d}{dx}(-2x) \cdot e^{-x^2} + (-2x) \cdot \frac{d}{dx}e^{-x^2} \quad \text{product rule}$$

$$= -2 \cdot e^{-x^2} - 2x \cdot \frac{d}{dx}(x^2) \cdot e^{-x^2}$$

$$= -2 \cdot e^{-x^2} - 2x \cdot (2x) \cdot e^{-x^2}$$

$$= (-2 + 4x^2) e^{-x^2}$$

$$f''(x) = 2(2x^2 - 1) \cdot e^{-x^2}$$

Summary:

$$f(x) = e^{-x^2}$$

$$f'(x) = -2x \cdot e^{-x^2}$$

$$f''(x) = 2(2x^2 - 1) \cdot e^{-x^2}$$