Graph $f(x) = e^{-x^2}$ by doing:

1. What do you know about $f$ as in general.

   Ans. $f$ is exponential with polynomial in exponent so is a smooth continuous function.

2. x-intercepts? None $(\ ,\ )$ $y=0$

   solve $e^{-x^2} = 0$ Has no solution $\Rightarrow$ No x-int

3. y-intercept: $(0, \ )$ $x=0$ $y=?$ $\boxed{y\text{-int}: (0,1)}$

   Set $x=0 \Rightarrow y = e^{-0^2} = e^0 = 1$

4. Critical Numbers: set $f'(x)=0$ solve for $x$

   $f'(x) = -2x \cdot e^{-x^2} = 0 \Rightarrow x=0$ (note: $e^{-x^2}$ is never 0)

   $f'(x)$ is defined everywhere so $x=0$ is only CN.

5. Critical Points: only one CP $(0, f(0))$

   $f(0) = e^{-0^2} = e^0 = 1 \Rightarrow$ $\boxed{(0,1) \text{ is only CP}}$

6. Use 2nd derivative test to determine what kind CP

   $f''(0) \overset{x=0}{=} 2(2(0)^2 - 1)e^{-0^2} = 2(0-1)\cdot e^0 = (+)(-)(+) = (-)$

   $\Rightarrow$ $\boxed{\text{CP } (0,1) \text{ is a Relative Maximum}}$

7. End Behavior: $f(x)$ will either have a HA or go to $\pm\infty$ as $x \to +\infty$ or $-\infty$

   Right End: $x \to +\infty$, $f(x) \to ?$

   $\lim_{x\to+\infty} e^{-x^2} = \lim_{x\to+\infty} \frac{1}{e^{x^2}} \Rightarrow \frac{1}{e^{\infty^2}} \to \frac{1}{e^{\infty}} \to \frac{1}{\infty} \to 0^+$

   $\Rightarrow$ at right end there is a horizontal asymptote given by $\lim y = 0$ and $f(x)$ approaches the HA from above.

   
   

   Left End: $x \to -\infty$, $f(x) \to ?$

   $\lim_{x\to-\infty} e^{-x^2} = \lim_{x\to-\infty} \frac{1}{e^{x^2}} \Rightarrow \frac{1}{e^{(-\infty)^2}} \to \frac{1}{e^{+\infty}} \to \frac{1}{+\infty} \to 0^+ \Rightarrow$ HA at $y=0$ from above