

Summary of End Behavior:



8. Vertical Asymptotes? Check to see if any VAs. VAs occur where functions are undefined and go to $\pm\infty$ at some value of x .

here $f(x) = e^{-x^2}$ (an exponential with polynomial in) is a smooth continuous function and is defined everywhere. Therefore, it has no vertical asymptotes. (It is a good habit to check for VAs)

9. Inflection Points: Set $f''(x) = 0$ ^{solve for x} for possible IPs. Check sign of $f(x)$ on both sides of x to check if IP.

solve

$$f''(x) = 2(2x^2 - 1) \cdot e^{-x^2} = 0 \quad e^{-x^2} \text{ is never zero so}$$

$$f''(x) = 0 \text{ is } 2x^2 - 1 = 0 \Rightarrow x^2 = \frac{1}{2}$$

$$\Rightarrow x = \pm \frac{1}{\sqrt{2}}$$

$\Rightarrow$ two possible IPs. one at $x = \frac{1}{\sqrt{2}}$ and one at $x = -\frac{1}{\sqrt{2}}$ must check both.

let check $x = -\frac{1}{\sqrt{2}} \approx -0.707 \approx -0.71$ need to check left/right

$$\text{left } f''(-.8) = 2(2(-.8)^2 - 1) \cdot e^{-(-.8)^2} = (+)(2(.64) - 1) \cdot (+) = (+)(+1.28) \cdot (+) = (+)$$

$$\text{right } f''(-.7) = 2(2(-.7)^2 - 1) \cdot e^{-(-.7)^2} = (+)(2(.49) - 1) \cdot (+) = (+)(.98 - 1) \cdot (+) = (-)$$

$$\Rightarrow f''(x) \begin{array}{c} (+) \\ -0.8 \\ \frac{1}{\sqrt{2}} \\ (-.707) \end{array} \quad \begin{array}{c} (-) \\ -\frac{1}{\sqrt{2}} \\ -.71 \end{array} \Rightarrow \text{IP at } x = -\frac{1}{\sqrt{2}} \approx -.71$$

$$f\left(-\frac{1}{\sqrt{2}}\right) = e^{-\left(-\frac{1}{\sqrt{2}}\right)^2} = e^{-\frac{1}{2}} = +.606$$

$$\text{IP: } \left(-\frac{1}{\sqrt{2}}, f\left(-\frac{1}{\sqrt{2}}\right)\right) = \left(-\frac{1}{\sqrt{2}}, e^{-\frac{1}{2}}\right) \approx (-.71, .6) \text{ IP}$$

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