MATH 220 **SAMPLE EXAM I** **update 6/2006**

Directions: Answer all questions and show all work in the **exam booklet** provided. Start each new question at the **top** of a new page and **box** your final answer. Questions 2 - 4, 6 are worth 15 points, Questions 1 and 5 are 20 points.

1. (a) Find the general solution of: $4y'' - 4y' + y = 0$
   (b) Find the general solution of: $y'' + 2y' + 2y = 0$
   (c) The functions x^2 and $1/x$ are solutions to a 2nd order, linear homogeneous ODE on $x > 0$. Verify whether or not the two solutions form a fundamental solution set.

2. Solve the IVP:
$$y\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2y + 1}, \quad y(0) = -1$$

3. Solve the IVP:
$$\frac{dy}{dt} + 4y = e^{-t}, \quad y(0) = 4/3$$

4. (a) Solve: $\dfrac{dy}{dx} = x(1 - y), \quad y(0) = 2$
   (b) Sketch the direction field corresponding to the ODE in part (a) for $x \geq 0$ and $0 \leq y \leq 2$.

5. A large tank is initially empty. At time $t = 0$, a brine solution begins to enter the tank at the rate of 6 L/min with concentration 2 Kg/L. The well-stirred solution is removed from the tank at the rate of 5 L/min. *State* and *solve* for $A(t)$ = the amount of salt in the tank at time t.

6. Consider $y' = -xy + y^2, \ y(0) = 1$
   (a) Approximate $y(1/10)$ using Euler's method with step size $h = 1/10$.
   (b) Approximate $y(1/10)$ using Improved Euler's method with step size $h = 1/10$. Use part (a) as your prediction step.