Topics for Math 220 - Fall 2002

- Chapter 1.
  Solutions and Initial Values
  Approximation via the Euler method
- Chapter 2.
  First Order: Linear
  First Order: Separation of Variables
  First Order: Integrating Factors
- Chapter 3.
  Modeling: "One Tank" Problems
  Modeling: Newtonian Mechanics
- Chapter 4.
  Second Order: Mass-Spring Oscillator
  Second Order: Fundamental Solutions & Wronskian
  Second Order: Linear (Real & Complex Roots)
  Second Order: Undetermined Coefficients
  Second Order: Variation of Parameters
- Chapter 5.
  Systems of Equations: "D method" or Elimination
  "Two Tanks" Problems
- Chapter 7:
  Laplace Transforms & Partial Fractions
  Laplace Transforms: Solving IVP
- Chapter 8.
  Taylor Polynomial Solutions
  Power Series Solutions
- Chapter 10.
  Heat Flow & Separation of Variables
  Boundary Value Problems
  Fourier Series
  Sine and Cosine Series
  Solving the Heat Equation with IVP

Sample problems from previous hour and final exams are grouped by topic below.

To make a "sample final" select 8 problems from this list, then work them without the help of your book or friends.

When you are done, compare problems with friends or after a drink, and decide which are easy for you, and which are hard.

You probably should then do at least one more of the easy ones, and two more of the hard ones!

The exam is at 6 PM Thursday, so <u>GET A GOOD NIGHT'S SLEEP WEDNESDAY NIGHT</u>!!!

All of your studying is wasted if you snooze through the test.

First Order – All methods

Problem 1: Obtain the solution to the following initial value problem showing all work:

$$y'(x) + (\sin x)\, y(x) = \sin x, \quad y(0) = 1$$

Problem 2: Obtain the solution to the following initial value problem showing all work:

$$x'(t) = e^{x(t)} \cos t - e^{x(t)}, \quad x(1) = 0$$

Problem 3: Construct the solution $y(x)$ of the initial value problem:

$$\frac{dy}{dx} = (x-3)(y+1)^{2/3}\ ,\ y(6) = 7$$

Problem 4: Solve the initial value problem

$$y' + \frac{2}{x} y = \frac{\cos x}{x^2}, y(\pi) = 0, x > 0$$

Problem 5: Solve the initial value problem:

$$y' + y^2 \sin x = 0\ ,\ y(\pi) = 1$$

Problem 6: Solve the IVP:

$$\frac{dy}{dt} = \frac{t}{e^{-y}} - \frac{e^y}{t}$$

1. State the name of the method you are using.
2. Find a solution.
3. Find a solution which satisfies the initial condition $y(1) = 1$.

Problem 7: Solve the IVP:

$$\frac{1}{x}\frac{dy}{dx} + y = x, y(1) = 1$$

Problem 8: Find the general solution of $-2xy' - y = x^3 - x$.

Problem 9: Find the solution of $y' + 2xy = x$ satisfying $y(0) = 0$.

Problem 10: Solve the IVP:

$$y\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2y + 1}, \quad y(0) = -1$$

Problem 11: Solve the IVP:

$$\frac{dy}{dt} + 4y = e^{-t}, \quad y(0) = 4/3$$

Problem 12: Solve: $\dfrac{dy}{dx} = x(1-y), \quad y(0) = 2$

Problem 13: Construct the solution of the Initial Value Problem
$$\frac{dy}{dx} - \frac{4}{x}y = x^3 + x^4e^{-x},\ y(1) = 0$$

Problem 14: Solve
$$\frac{dy}{dx} - \frac{y-1}{x^2y} = 0$$

Problem 15: Find the general solution and the solution of the IVP
$$\frac{dy}{dx} - \frac{4}{x}y = x^3 + x^4e^{-x},\ y(1) = 0$$

Problem 16: Find the general solution and the solution of the IVP
$$xy'(x) + 3y(x) = x^2 + x - 1,\ y(1) = -1/20$$

Problem 17: Find the general solution and the solution of the IVP
$$y' = 3y^2 - xy^2,\ y(0) = 1$$

Euler Method

Problem 1: Given the ODE:
$$\frac{dy}{dt} = f(t,y) \text{ where } f(t,y) = \ln(t+y)$$

1. State Euler's numerical algorithm for this equation using step size $h = 1/10$.
2. Given that $y(1) = 1$, compute y(1), y(1.1) and y(1.2) in the case when $f(y,t) = \ln(t+y)$.

Problem 2: Given the ODE:
$$\frac{dy}{dt} = f(t,y) \text{ where } f(t,y) = -ty + y^2$$

1. State Euler's numerical algorithm for this equation using step size $h = 1/10$.
2. Given that $y(1) = 1$, compute $y(1)$ and $y(1.1)$ in the case when $f(t,y) = -ty + y^2$.

Problem 3: Consider $y' = -xy + y^2,\ y(0) = 1$

1. Approximate $y(1/10)$ using Euler's method with step size $h = 1/10$.
2. Approximate $y(1/10)$ using Improved Euler's method with step size $h = 1/10$. Use part (a) as your prediction step.

Second Order, Homogeneous & Undetermined Coefficients

Problem 1: Consider the following equation for $y(x)$:

$$y'' + 4y' + 5y = 5x$$

a) Find a fundamental set of solutions to the corresponding homogeneous equation.

b) Construct a particular solution.

c) Give the general solution.

Problem 2: Find the solution to the equation $y'' - 2y' + y = 0$ satisfying the initial conditions $y(0) = 1$ and $y'(0) = -2$

Problem 3: a) Find two fundamental solutions to the equation $y'' + y' + y = 0$.

b) Find all solutions to the equation $y'' + y' + y = 2\cos(2x)$

Problem 4: a) Find two fundamental solutions to the equation $y'' + 5y' + 6y = 0$ and compute their Wronskian.

b) Find all solutions to the equation $y'' + 5y' + 6y = \sin x$.

Problem 5: Find the solution to the equation $x'' - 2x' + 10x = 0$ satisfying the initial conditions $x(0) = 1$ and $x'(0) = -5$.

Problem 6: Find the general solution of: $4y'' - 4y' + y = 0$

Problem 7: Find the general solution of: $y'' + 2y' + 2y = 0$

Problem 8: Consider the following equation for $y(x)$:

$$y'' + 2y' = 6x$$

a) Find a fundamental set of solutions to the corresponding homogeneous equation

b) Construct a particular solution.

c) Give the general solution.

Problem 9: Consider the following equation for $y(x)$:

$$y'' + y' = e^{-x}$$

a) Find a fundamental set of solutions $y_1(x)$ and $y_2(x)$ to the corresponding homogeneous equation

b) Construct a particular solution using the *method of undetermined coefficients*

c) Give the general solution.

Problem 10: Consider the following equation for $y(x)$:

$$y''(x) + 4y' + 4y(x) = \sin(\pi x)$$

1. Find a fundamental set of solutions $y_1(x)$ and $y_2(x)$ to the corresponding homogeneous equation.
2. Find the Wronskian of the solution set.
3. Construct a particular solution using the *method of undetermined coefficients.*
4. Find the solution which satisfies the initial conditions $y(0) = 1$ and $y'(0) = 0$.

Second Order, Vibrating Springs

Problem 1: A vibrating spring with damping is modeled by the differential equation

$$y'' + 2y' + 4y = 0.$$

1. Find the general solution to the equation. Show each step of the process.
2. Is the solution under damped, over damped or critically damped?
3. Suppose that the damping were changed, keeping the mass and the spring the same, until the system became critically damped. Write the differential equation which models this critically damped system. *Do not solve.*
4. What is the steady state (long time) solution to

$$y'' + 2y' + 4y = \cos(2t).$$

Second Order, Variation of Parameter

Problem 1:

1. Find the general solution of: $y''(x) + 2y'(x) + 4y(x) = 0$ and compute the Wronskian of the solution set.
2. Find the solution to

$$y''(x) - 9y(x) = -e^x + x^2 \quad y(0) = 0, \;\; y'(0) = 0.$$

Problem 2:

1. Find the general solution of: $y''(x) + 2y'(x) + y(x) = 0$.
2. Find the solution to

$$y''(x) - 4y(x) = -e^x + x \quad y(0) = 0, \;\; y'(0) = 0.$$

Problem 3: Find the general solution of $y'' + 16y = \sec(4x)$

Second Order, Wronskian & Fundamental Solution Sets

Problem 1: The functions x^2 and $1/x$ are solutions to a 2nd order, linear homogeneous ODE on $x > 0$. Verify whether or not the two solutions form a fundamental solution set.

Mixing Tanks

Problem 1: A tank contains 1000 L of water (with no salt initially) into which a solution of salt, with concentration of 2 kg/L is added at the constant rate of 6 L/min. The solution is well-stirred and i flowing out of the tank at the rate of 5L/min. Give the IVP for the amount of salt $A(t)$ in the tank at time t. Solve the IVP and determine $A(2)$.

Problem 2 Consider a large tank holding 1,000 gallons of brine solution, initially containing 10 lbs of salt. At time $t = 0$, more brine solution begins to flow into the tank at the rate of 3 gal/min. The concentration of salt in the solution entering the tank is e^{-t} lbs/gal, i.e. varies in time. The solution inside the tank is well-stirred and is flowing out of the tank at the rate of 5 gal/min. Solve for $A(t)$ = the amount of salt in the tank (in lbs.) at time t.

Problem 3: Consider a large tank holding 2,000 gallons of brine solution, initially containing 10 lbs of salt. At time $t = 0$, more brine solution begins to flow into the tank at the rate of 2 gal/min. The concentration of salt in the solution entering the tank is $3e^{-t}$ lbs/gal, i.e. varies in time. The solution inside the tank is well-stirred and is flowing out of the tank at the rate of 5 gal/min. Solve for $A(t)$ = the amount of salt in the tank (in lbs.) at time t:

Problem 4: A nitric acid solution flows at a constant rate of 6 L/min into a large tank that initially held 100 L of a 0.2% nitric acid solution. The solution inside the tank is kept well stirred and flows out of the tank at the same rate. If the solution entering the tank is 50% nitric acid, determine the volume of nitric acid in the tank after t minutes.

Problem 5: A large tank is filled with 40 Liters of pure water. A pipe runs into the tank, and carries in salt water at the rate of 5 L/min. The incoming water has .2 kg/L of salt in it. The pipe also has an outlet valve on the bottom, where water drains at the rate of 5 L/min.

a) Assume the water in the tank is well-mixed, give a differential equation for the amount of salt $x(t)$ in the tank at time t.

b) How much salt is in the tank after 8 minutes?

Problem 6: A large tank is initially empty. At time $t = 0$, a brine solution begins to enter the tank at the rate of 6 L/min with concentration 2 Kg/L. The well-stirred solution is removed from the tank at the rate of 5 L/min. Give the differential equation which $A(t)$ satisfies, then and solve the equation for $A(t)$ = the amount of salt in the tank at time t.

Problem 7: Consider a tank holding 1,000 gallons of brine solution, initially containing a concentration of 1lbs/gal of salt. At time $t = 0$, more brine solution begins to flow into the tank at the rate of 3 gal/min. The concentration of salt in the solution entering the tank is 0.2 lbs/gal. From time $t = 3$ forward the flow into the tank continues at 3 gal/min but the salt concentration drops off according to the formula $0.2e^{-2(t-3)}$ where t is time measured *from* $t = 0$. The solution inside the tank is well-stirred and is flowing out of the tank at the rate of 6 gal/min.

Set up **but do not solve** the problem for $A(t)$ = the amount of salt in the tank (in lbs.) at time t.

Problem 8: Mixing tank A holds 100 L of liquid, and mixing tank B holds 50 L of liquid. The tanks are interconnected by pipes with the liquid flowing from tank A into tank B at a rate of 7 L/min and from B into A at a rate of 2 L/min. The liquid inside of each tank is kept well stirred. A brine solution with a concentration of .5 kg/L of salt flows into A at a rate of 6 L/min. The (diluted) solution flows out of the system from tank A at 1 L/min and from tank B at 5 L/min. Initially, tank A contains pure water (0 kg salt) and tank B contains 5 kg of salt.

Set up **but do not solve** the system of coupled differential equations satisfied by the variables $x(t) =$ the amount of salt in tank A, and $y(t) =$ amount of salt in tank B.

Problem 9: Two mixing tanks (A and B) each holding 100 L of liquid are interconnected by pipes with the liquid flowing from tank A into tank B at a rate of 3 L/min and from B into A at a rate of 2 L/min. The liquid inside of each tank is kept well stirred. A brine solution with a concentration of 6 kg/L of salt flows into A at a rate of 2 L/min. The (diluted) solution flows out of the system from tank A at 1 L/min and from tank B at 1 L/min.

1. Set up the coupled ODE's for the amount of salt in each tank. Hint: the volumes of both tanks will turn out to be constant.
2. If initially tank A contains pure water and tank B contains 200 kg of salt, determine the mass of salt in each tank at time $t \geq 0$. begin

Problem 10: Consider a system of two well-stirred, interconnected tanks that are filled to capacity. Tank 1 holds 5 L of water, initially containing 2 Kg of salt. Tank 2 has a capacity of 4 L and initially is filled with fresh water. Fresh water flows into tank 1 at the rate of 8 L/min while salt solution flows out of tank 2 at the same rate. Salt solution is pumped from tank 1 to tank 2 at the rate of 20 L/min and salt solution is pumped from tank 2 to tank 1 at the rate of 12 L/min.

Set up the system of differential equations with initial conditions modeling the amount of salt $A(t)$ in tank 1 and amount of salt $B(t)$ in tank 2.

Solve the system of equations for $A(t)$ and $B(t)$.

Problem 11: A 20 L tank is initially filled with fresh water. Fluid leaves the tank from the bottom at the rate of 10 L/min and water enters the tank from the top at the same rate. An accident occurs at $t = 0$ and salt contaminates the incoming water, causing the water entering the tank to have salt concentration 1kg/L. At time $t = 5$, the error is discovered and the source of salt is stopped so that the entering water is again fresh. Find $x(t)$ which is the amount of salt in the tank at time t.

Systems of Equations

Problem 1: Use the method of substitution to find the solutions $x(t)$ and $y(t)$ for the system of equations:

$$x'(t) + y(t) = 1, \;\; x(0) = 1,$$
$$x(t) + y'(t) = t, \;\; y(0) = 0.$$

Problem 2: Find the solution $x(t)$ and $y(t)$ of the system

$$\begin{aligned} x' + x + y &= 1 \\ -2x + y' - y &= 8 \end{aligned}$$

with initial conditions $x(0) = -8$ and $y(0) = 9$.

Boundary Value

Problem 1: Consider the boundary value problem:

$$y'' + Ay = 0, \quad 0 < x < \pi,$$
$$y(0) = 0; \quad y'(\pi) = 0$$

Find the *smallest* value of $A > 0$ such that the BVP has a nonzero solution.

Problem 2: Consider the boundary value problem:

$$y'' + Ay = 0, \quad 0 < x < 1,$$
$$y'(0) = 0; \quad y(1) = 0$$

Find the *smallest* value of $A > 0$ such that the BVP has a nonzero solution.

Problem 3: Consider the boundary value problem:

$$y''(x) + Ky(x) = 0, \quad -1 < x < 1,$$
$$y(-1) = 0; \quad y(1) = 0$$

Find the **smallest** value of $K > 0$ such that the BVP has a nonzero solution.

Problem 4: Consider the boundary value problem:

$$y'' + 4y = 0, \; 0 < x < L; \; y'(0) = 0; y(L) = 0$$

Find the smallest value of $L > 0$ such that the BVP has nonzero solution.

Problem 5: Consider the boundary value problem:

$$y'' + y = 0, \; 0 < x < L; \; y'(0) = 0; y(L) = 0$$

Find the smallest value of $L > 0$ such that the BVP has nonzero solution.

Laplace Transform

Problem 1: Solve the given initial value problem using the method of Laplace transform. No credit will be given for any other method.

$$y'' - 4y' + 5y = 4e^{3t}, \qquad y(0) = 2, \quad y'(0) = 7.$$

Problem 2: Find $Y(s)$ the Laplace transform and then $y(t)$ for the initial value problem:

$$y' + 6y = u(t-3), \quad y(0) = 1$$

Problem 3: Solve the initial value problem using the method of Laplace Transform. Show all your work, including details of work to solve the partial fraction expansion.

$$y'' - 4y' + 5y = 0 \ ; \ y(0) = 2 \ ; \ y'(0) = 3$$

Problem 4: Find $Y(s)$ the Laplace transform and then $y(t)$ for the initial value problem:

$$y' + 6y = [u(t-3) - u(t-2)]e^{-6(t-3)}, \quad y(0) = 1$$

Problem 5: Use the method of partial fractions and the table on the reverse side to find the inverse Laplace Transform $y(t) = \mathcal{L}^{-1}\{F(s)\}$ of

$$F(s) = \frac{5}{(s^2+4)(s+1)}$$

Problem 6: Find the Laplace transform $F(s)$ of the solution $y(t)$ of the differential equation with time delay

$$y'(t) + 3y(t) = 1 - \cos(t)u(t-\pi) \ ; \ y(0) = 1$$

Problem 7: Find $Y(s)$ the Laplace transform and then $y(t)$ for the initial value problem:

$$y'(t) - 3y(t) = u(t-1), \quad y(0) = 1$$

Problem 8: Use the method of partial fractions and the table on the reverse side to find the inverse Laplace Transform $y(t) = \mathcal{L}^{-1}\{F(s)\}$ of

$$F(s) = \frac{5}{(s^2+4)(s+1)}$$

Problem 9: Find the **Laplace transform** $Y(s)$ as well as the solution $y(t)$ to:

$$y'(t) + 3y(t) = u(t-2) \ , \ y(0) = 1$$

Problem 10: Use the method of the Laplace Transform to solve for $y(t)$

$$y'' + 4y' + 4y = u(t-2), \ y(0) = 1, y'(0) = 0$$

Problem 11: Apply the Laplace Transform to both sides of the differential equation in the initial value problem below, solve for $Y(s) = \mathcal{L}\{y(t)\}(s)$ and then apply the inverse Laplace Transform to solve for $y(t)$.

$$y' - 4y = u(t - 0.5),\ y(0) = 2$$

Problem 12: Find the Laplace Transform $Y(s) = \mathcal{L}\{y(t)\}$:

$$y'' + 4y' + 8y = \sin(2t) + (t-1)^4,\ y(0) = 1, y'(0) = 0$$

Taylor Polynomial

Problem 1: Determine the Taylor polynomial of degree 2 for the solution to the initial value problem

$$y' = \frac{1}{2x + 3y - 1}, \quad y(0) = 2.$$

Problem 2: Find the first three non-zero terms in the Taylor series expansion of $y(t)$ about $t = 0$ for the solution to

$$y'(t) - (t+1)[y(t)]^2 = 0\ ;\ y(0) = 3$$

Problem 3: Find the first three non-zero terms in the Taylor series solution about $x = 0$ to

$$y'(t) + 2(t-1)y(t) = 0,\ \ y(0) = -1$$

Problem 4: Find the first three non-zero terms in the Taylor series expansion of $y(t)$ about $t = 0$ for the solution to

$$y'(t) + (t+1)[y(t)]^2 = 0 \text{ with initial condition } y(0) = 1.$$

Problem 5: Find the first three non-zero terms in the Taylor series solution about $x = 0$ to

$$y'(t) + 2(t+3)y(t) = 0,\ \ y(0) = 1$$

Problem 6: Find the first three non-zero terms in the Taylor series solution about $t = 0$ to

$$y'(t) + (t-1)(y(t))^2 = 0,\ y(0) = 1$$

Problem 7: Find the first three non-zero terms in the Taylor series solution about $t = 0$ to

$$y'(t) + 3(t+2)y(t) = 0,\ y(0) = 5$$

Power Series

Problem 1: Find the first three non-zero terms in the power series solution to

$$y'(t) + 3(t+3)y(t) = 0, \quad y(0) = 1$$

Problem 2: Find the first 4 non-zero terms in the series solution to $y'' - xy = 0$ about $x = 0$ such that $y(0) = 1$ and $y'(0) = -1$.

Fourier Series

Problem 1: Consider

$$f(x) = \begin{cases} \frac{1}{2}, & 0 < x < \pi/2, \\ -1, & \pi/2 < x < \pi. \end{cases}$$

1. Sketch the **odd** extension of $f(x)$ to the interval $-\pi < x < \pi$.
2. Sketch the **even** extension of $f(x)$ to the interval $-\pi < x < \pi$.

Problem 2: Consider

$$f(x) = \begin{cases} 0, & 0 < x < \pi/2, \\ -1, & \pi/2 < x < \pi. \end{cases}$$

1. Sketch the **odd** extension of $f(x)$ to the interval $-\pi < x < \pi$.
2. Sketch the **even** extension of $f(x)$ to the interval $-\pi < x < \pi$.

Problem 3: Consider

$$f(x) = \begin{cases} 1, & 0 < x \le \pi/2, \\ 2, & \pi/2 < x \le \pi. \end{cases}$$

1. Extend as an *ODD* function on $-\pi < x < \pi$ and sketch a picture of the function over this interval.
2. Find the first three non-zero terms of the Fourier series for the odd extension of $f(x)$ which represents the function on the interval $[-\pi, \pi]$. (That is, find the Fourier sine series for $f(x)$ on $0 \le x \le \pi$.)

Problem 4: Construct a Fourier sine series for $f(x) = x$ for $0 < x < \pi$.

Problem 5: Consider $f(x) = \begin{cases} 0, & -\pi/2 \le x \le \pi/2 \\ 1, & \pi/2 < x < \pi \end{cases}$

Find the first three non-zero terms of the Fourier sine series for $f(x)$ for $0 < x < \pi$.

Problem 6: Consider $f(x) = \begin{cases} -1/2, & 0 \le x < \pi/2, \\ 1, & \pi/2 \le x < \pi. \end{cases}$

a) Sketch the odd extension of $f(x)$ to the interval $-\pi < x < \pi$

b) Find the first three non-zero terms of the Fourier sine series for $f(x)$

Heat Equation

Problem 1: Using separation of variables and Fourier method, find the solution $u(x,t)$ of the heat conduction problem:

$$\begin{gathered}\frac{\partial u}{\partial t} = 5\frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < \pi, \quad t > 0,\\ u(0,t) = u(\pi,t) = 0, \qquad t > 0,\\ u(x,0) = (\pi - x), \qquad 0 < x < \pi\end{gathered}$$

Problem 2: Find the solution $u(x,t)$ of the heat conduction problem

$$\begin{gathered}\frac{\partial u(x,t)}{\partial t} = 2\frac{\partial^2 u(x,t)}{\partial x^2}, \quad 0 < x < 1, \ \ t > 0;\\ \frac{\partial u(0,t)}{\partial x} = 0, \quad \frac{\partial u(1,t)}{\partial x} = 0, \quad t > 0;\\ u(x,0) = \pi^2 + 7\cos \pi x - 8\cos 100\pi x, \quad 0 < x < 1\end{gathered}$$

Problem 3: Find the solution $u(x,t)$ of the heat conduction problem

$$\begin{gathered}\frac{\partial u(x,t)}{\partial t} = 2\frac{\partial^2 u(x,t)}{\partial x^2}, \quad 0 < x < 1/2, \ \ t > 0;\\ \frac{\partial u(0,t)}{\partial x} = 0, \quad \frac{\partial u(1/2,t)}{\partial x} = 0, \quad t > 0;\\ u(x,0) = 10 - 3\cos(10\pi x) + \pi\cos(-2\pi x), \quad 0 < x < 1/2\end{gathered}$$

What is the steady state solution to this problem? I.e. what is $lim_{t\to\infty} u(x,t)$?

Problem 4: Find the solution $u(x,t)$ of the heat conduction problem

$$\begin{gathered}\frac{\partial u(x,t)}{\partial t} = 4\frac{\partial^2 u(x,t)}{\partial x^2}, \quad 0 < x < 2\pi, \ \ t > 0;\\ \frac{\partial u(0,t)}{\partial x} = 0, \quad \frac{\partial u(2\pi,t)}{\partial x} = 0, \quad t > 0;\\ u(x,0) = 10 - 3\cos 2x + 5\cos(3x), \quad 0 < x < 2\pi\end{gathered}$$

Find the steady state solution for $u(x,t)$, i.e. $\lim_{t\to\infty} u(x,t)$.

Problem 5: Find the solution $u(x,t)$ of the heat conduction problem

$$\begin{gathered}\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < 3, t > 0\\ U(0,t) = 0, u(3,t) = 0, t > 0\\ u(x,0) = 6\sin(\pi x/3) + 4\sin(\pi x) - 7\sin(4\pi/3), 0 < x < 3\end{gathered}$$

Problem 6: Find the solution $u(x,t)$ of the heat conduction problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0$$
$$u_x(0,t) = 0, u_x(1,t) = 0, t > 0$$
$$u(x,0) = \pi - 3\cos(6\pi x) + \pi\sin(4\pi x), 0 < x < 1$$

What is the steady state solution to this problem? i.e., what is $\lim_{t\to\infty} u(x,t)$?

Problem 7: Find the solution $u(x,t)$ of the heat conduction problem:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < \pi, t > 0$$
$$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0, t > 0$$
$$u(x,0) = 4 - 2\cos 3x + 7\cos 4x, 0 < x < \pi$$

Problem 8: Find the solution $u(x,t)$ of the heat conduction problem:

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}, 0 < x < \pi, t > 0$$
$$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0, t > 0$$
$$u(x,0) = \pi + 2\cos x - 7\cos 3x, 0 < x < \pi$$