Math 220: SAMPLE FINAL EXAM II - revised 11.26.06

1. Consider the following equation for $y(x)$:

$$y'' + 2y' = 6x$$

   (a) Find a fundamental set of solutions to the corresponding homogeneous equation.

   (b) Construct a particular solution.

   (c) Give the general solution.

2. Construct the (implicit) solution $y(x)$ of the initial value problem:

$$e^y \frac{dy}{dx} = (\sin x)(e^y + 3)^{1/2}, \quad y(0) = 0$$

3. (a) Find the general solution of: $x^2y'' - xy' + y = 0, \quad x > 0$

   (b) Solve:

$$\begin{aligned} x' &= 3y \\ y' &= 2x - y. \end{aligned}$$

4. Consider the boundary value problem:

$$y'' + 4y = 0, \quad 0 < x < L; \quad y'(0) = 0; \quad y(L) = 0$$

   Find the smallest value of $L > 0$ such that the BVP has a nonzero solution.

5. (a) Find the Laplace transform $Y(s) = \mathcal{L}\{y(t)\}$:

$$y'' + 4y' + 8y = \sin 2t + (t-1)^4, \quad y(0) = 1, \ y'(0) = 0$$

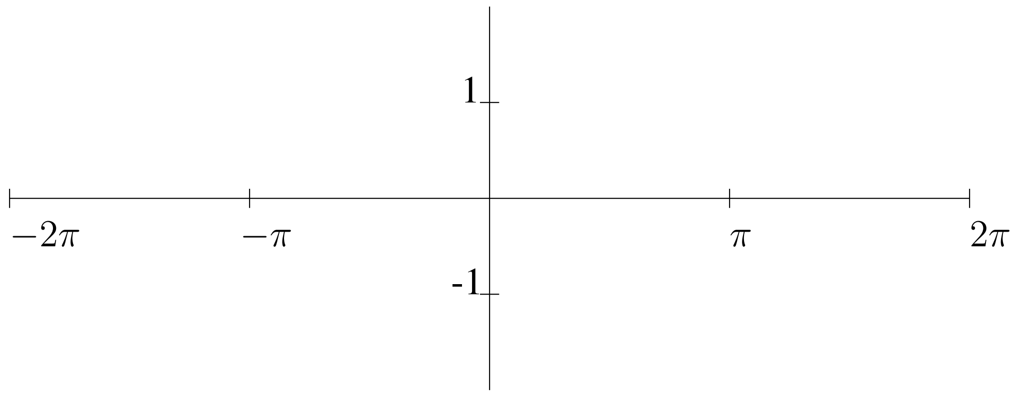
   (b) Let $G(s) = \mathcal{L}\{g(t)\}$. Express as a function of t (without *explicitly* computing constants), the inverse Laplace transform
   $\mathcal{L}^{-1}\left\{\dfrac{8}{s^3(s^2 - s - 2)} + \dfrac{G(s)}{s^2+1}\right\} =$

6. Consider

$$f(x) = \begin{cases} 0, & 0 < x < \pi/2 \\ 1, & \pi/2 < x < \pi \end{cases}$$

(a) Construct a Fourier cosine series for $f(x)$.

(b) Sketch a graph showing the values the series in (a) converges to on $-2\pi < x < 2\pi$.



7. A 20L transfer tank is initially filled with fresh water. Fluid leaves the tank from the bottom at the rate of 10 L/min and water enters the tank from the top at the same rate. An accident occurs at $t = 0$ and salt contaminates the incoming water causing the water entering from the top to have a salt concentration of 1 kg/L. At time $t = 5$, the error is discovered and the source of salt is stopped so that the entering water is again fresh. Find $x(t)$ which is the amount of salt in the tank at time t.

8. Find the solution $u(x,t)$ of the heat conduction problem

$$u_t = u_{xx}, \quad 0 < x < \pi, \;\; t > 0$$
$$u_x(0,t) = 0 \quad u_x(\pi,t) = 0, \quad t > 0$$
$$u(x,0) = 4 - 2\cos 3x + 7\cos 4x, \quad 0 < x < \pi,$$