Math 550 – Homework 2

For discussion: Week 3 (Wednesday 5-6 pm in SEO 427).

Reminder: Aim to solve at least 3 of the following problems. If you can regularly attend discussion session, please do so and volunteer to present the problems you have solved. If you have a recurring conflict with discussion sessions, please let me know and we will make an arrangement.

The following problems from Lee's book, Chapter 17: 5, 6, 7; and Chapter 18: 9.
For 17-6 and 7, only do the first part (the cases $0 \leq k \leq n-2$).

Problem 5: Use the Mayer-Vietoris sequence to compute the cohomology of real projective space. More precisely, prove that

$$H^k(\mathbb{R}P^n) = \begin{cases} \mathbb{R} & \text{if } k = 0 \\ \mathbb{R} & \text{if } k = n \text{ and } n \text{ is odd.} \\ 0 & \text{otherwise} \end{cases}$$

Problem 6: Use the Mayer-Vietoris sequence to compute the cohomology of T^2.

Problem 7 (Bott-Tu I.4.5): Prove that the "integration along the fiber" map π_* and the exterior derivative d commute.

Problem 8 (Bott-Tu I.4.8): Use the Mayer-Vietoris sequence to compute the cohomology groups $H^*(M)$ and $H^*_c(M)$ where M is the open Möbius strip.

Problem 9: Let M, N be closed connected oriented manifolds and let $y_0 \in N$. Compute the Poincaré dual of $M \times \{y_0\}$. If N is noncompact, how does this answer change (for the closed Poincaré dual? for the compact Poincaré dual?).

Problem 10 (Bott-Tu I.5.16): Set $X = \mathbb{R}^2 \backslash \{0\}$. Let x, y.be the standard coordinates and r, θ the polar coordinates on X.

(a) Prove that the Poincaré dual of the ray $\{(x, 0) \mid x > 0\}$ in X is $[d\theta/2\pi] \in H^1(X)$.

(b) Prove that the Poincaré dual of the unit circle is the nontrivial generator $\rho(r)dr \in H^1_c(X)$ where ρ is a bump function on $\mathbb{R}$ with total integral 1.