Math 550 – Homework 4

For discussion: 2/12 at 5-6 pm in SEO 427.

From Lee's book, Chapter 20: 1, 3, 5, 6, 7, 17, 20, 21, 22.

The last three of the above problems concern the adjoint representation of a Lie group, so here is a summary (see Lee's book, towards end of Chapter 20 for proofs of the assertions made here):

- For an element $g \in G$ of a Lie group, conjugation by g is a diffeomorphism $G \to G$ fixing identity, so its derivative $\mathrm{Ad}(g)$ is a linear automorphism of $\mathfrak{g}$. This means we have a (smooth) representation $\mathrm{Ad}: G \to \mathrm{GL}(\mathfrak{g}), g \mapsto \mathrm{Ad}(g)$.
- There is also a Lie algebra version: For $X \in \mathfrak{g}$, we have $\mathrm{ad}(X) : \mathfrak{g} \to \mathfrak{g}$ defined by $\mathrm{ad}(X)(Y) := [X, Y]$. This gives a Lie algebra morphism $\mathrm{ad}: \mathfrak{g} \to \mathrm{End}(\mathfrak{g})$.
- The relationship between these is that the derivative at identity of Ad is ad, i.e. $\mathrm{Ad}_* = \mathrm{ad}$.