Math 550 – Final project suggestions

Below is a list of suggestions for final topics. The list is limited by own knowledge and biases, and you should feel free to look for other topics (but please let me know what you are thinking about).

1. The only spheres that are Lie groups are S^1 and S^3.
2. Cohomology of compact Lie groups (Hopf's Theorem).
3. Construction of classifying space BG of a (compact, if you prefer) Lie group G.
4. Cohomology of BG (you can aim to show it vanishes in odd degrees; the strongest theorem here is that the Chern-Weil homomorphism is an isomorphism)
5. Seifert fiber spaces.
6. Thurston's classification of 3-dimensional geometries.
7. Closed 3-manifolds that fiber over the circle in infinitely many ways.
8. Existence of aspherical homology 3-spheres.
9. Homotopy equivalent but non-homeomorphic closed manifolds
10. Milnor's construction of exotic spheres. Or Brieskorn spheres.
11. Smale's theorem on existence of sphere eversions.
12. Tischler's theorem on foliations and fiber bundles over the circle.
13. Thurston's theorems on codimension 1 foliations.
14. Reeb-Thurston stability theorem.
15. Novikov's compact leaf theorem.
16. Haefliger's theorem on analytic codimension 1 foliations.
17. Plante's theorem on growth of leaves in codimension 1 foliations.
18. Characteristic classes of foliations.
19. Symplectic manifolds and Darboux's theorem.
20. Symplectic structure on the (infinite dimensional) space of connections on a fiber bundle (Atiyah-Bott).
21. Ambrose-Singer theorem that holonomy is generated by curvature.
22. De Rham holonomy theorem and/or Berger's classification of holonomies.

23. The Milnor-Wood inequality for circle bundles over surfaces.

24. Surface bundles are determined by mapping class group-valued representations (see Earle-Eells theorem).

25. Characteristic classes of surface bundles / cohomology of mapping class groups.

26. Thom's theorem on cohomology classes as integration over submanifolds (in its entirety it is very long and complicated; pick some part(s) to explain and black box the rest).

27. whatever you find interesting ;).