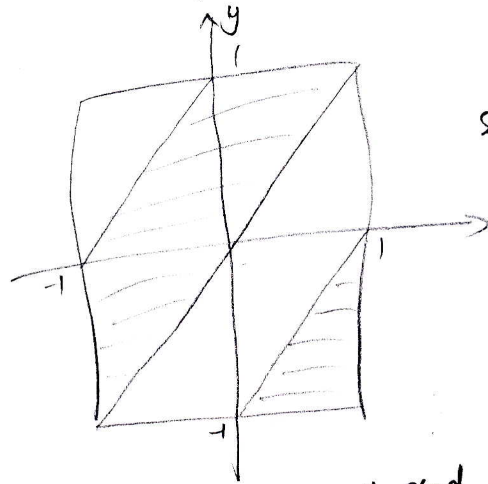
$\boxed{\text{Prob. } 5.3}$ cdf for $D = X - Y$. where $f(x,y) = \begin{cases} \frac{1}{2} & x,y \in S \\ 0 & \text{o.w} \end{cases}$

$D = X - Y$



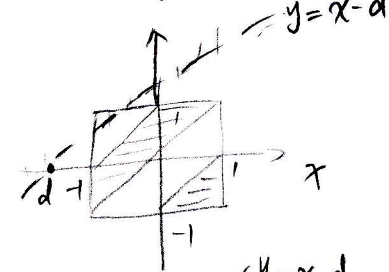
Support S

$$P(D \leq d) = P(X - Y \leq d)$$
$$= P(Y \geq X - d)$$
$$= \iint_{\substack{y \geq x-d \\ x,y \in S}} f(x,y)\, dx\, dy$$

Case 1. $d \leq -1$, integration region

$$\{y \geq x - d\} \cap S = \phi$$

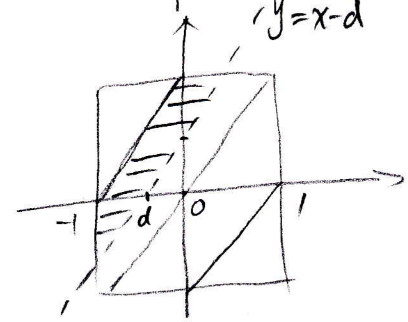
area $= 0$ $\Rightarrow P(D \leq d) = 0$



(case 1)

case 2. $-1 < d \leq 0$,

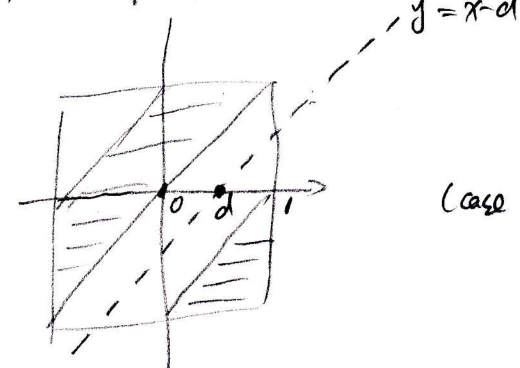
$$\text{area}(\{y \geq x - d\} \cap S)$$
$$= 2 \times (1+d)^2/2 + [\tfrac{1}{2} - \tfrac{1}{2}d^2]$$
$$= (1+d)^2 + \tfrac{1}{2}(1-d^2) = (1+d)(3+d)/2$$
$$P(D \leq d) = (1+d)(3+d)/4$$



(case 2)

case 3. $0 < d \leq 1$

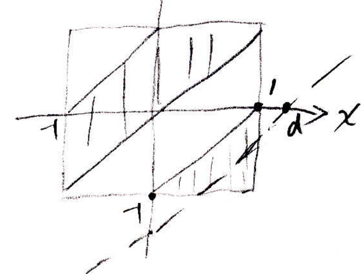
$$\text{area}(\{y \geq x - d\} \cap S)$$
$$= \text{area}(\{y \geq x\} \cap S) = \tfrac{3}{2}$$
$$P(D \leq d) = \tfrac{3}{2} \times \tfrac{1}{2} = \tfrac{3}{4}$$



(case 3)

Case 4. $1 < d \leq 2$

$$\text{area}(\{y \geq x - d\} \cap S)$$
$$= \tfrac{3}{2} + (\tfrac{1}{2} - \tfrac{1}{2}(2-d)^2) = 2 - (2 - 2d + \tfrac{d^2}{2}) = 2d - \tfrac{d^2}{2}$$
$$P(D \leq d) = (2d - \tfrac{d^2}{2}) \cdot \tfrac{1}{2} = d(4-d)/4$$



(case 4)

case 5. $d > 2$

$$\text{area}(\{y \geq x-d\} \cap S) = S, \quad P(D \leq d) = 1$$